Working Paper

Department of Finance

Economic and Fiscal Policy Branch

Working papers are circulated in the language of preparation, to make analytic work undertaken by the staff available to a wider readership. They have received only limited evaluation and do not reflect the views of the Department of Finance. Comments on the working papers are invited and may be sent to the author(s).

102-00408/

The Role of Government Debt in a World of Incomplete Financial Markets

Steven James and Philippe Karam

Department of Finance Working Paper
2001-01

This paper was written when both authors were members of Economic Studies and Policy Analysis Division of Finance Canada. The authors would like to thank Ellen McGrattan for providing us with code required to solve the model algorithm, and for very useful conversations. All errors or omissions are our own. The views expressed in this paper are those of the authors and do not reflect those of the Department of Finance.

CONTENTS

ABSTRACT 1

EXECUTIVE SUMMARY 2

1. INTRODUCTION 7

2. COMPLETE VERSUS INCOMPLETE MARKETS 8

2.1 THE STANDARD COMPLETE MARKETS FRAMEWORK 8
2.1.1 EMPIRICAL SHORTCOMINGS 9
2.2 THE INCOMPLETE MARKETS FRAMEWORK 10
2.2.1 THE ROLE OF GOVERNMENT DEBT 10
2.2.2 MODELS IN WHICH GROSS DEBT ENHANCES LIQUIDITY 10
2.2.3 A MODEL IN WHICH NET DEBT ENHANCES LIQUIDITY: THE AIYAGARI-MCGRATTAN MODEL 11
2.2.4 INTUITION UNDERLYING THE AM RESULT 13
2.2.5 FISCAL POLICY IN THE AM MODEL 14
2.2.6 ASSESSING THE AM MODEL 15

3. OUR APPROACH 16

3.1 ALTERNATIVE FISCAL DIVIDEND RULE 16
3.2 INTRODUCING A WELFARE ROLE FOR GOVERNMENT SPENDING 17
3.3 INTRODUCING OPEN ECONOMY FEATURES 18

4. CONCLUSIONS 22

BIBLIOGRAPHY 28

TECHNICAL APPENDIX 30

Abstract

Aiyagari and McGrattan (1998) developed a model in which optimal net government debt in the steady state is positive. This result arises from a particular form of financial market incompleteness: that households cannot buy insurance against prolonged spells of low income. In this paper we extend the Aiyagari-McGrattan model in ways that preserve incomplete financial markets but overturn the result that optimal net government debt is positive. In particular, we show that Aiyagari and McGrattan implicitly assume that government debt crowds out government waste, which provides an additional channel for net debt to provide social benefits. We show that when this channel is blocked, positive net debt is no longer optimal. We also relax the assumption that the economy is closed and show that in an open economy the "insurance" benefit provided by net government debt is diluted by the fact that foreigners will hold some of the debt. In our open economy case positive net debt is again no longer optimal.

Executive Summary

1. Introduction

The notion that net government debt is too high is generally accepted in the Canadian academic and policy community. It is supported by research that shows that debt is economically costly and imposes burdens on future generations. However, this research provides little guidance on the issue of how low the debt should ultimately go.

This paper draws on a new and rapidly evolving literature that assumes incomplete financial markets, which provides an important economic role for net government debt not found in standard complete markets models.

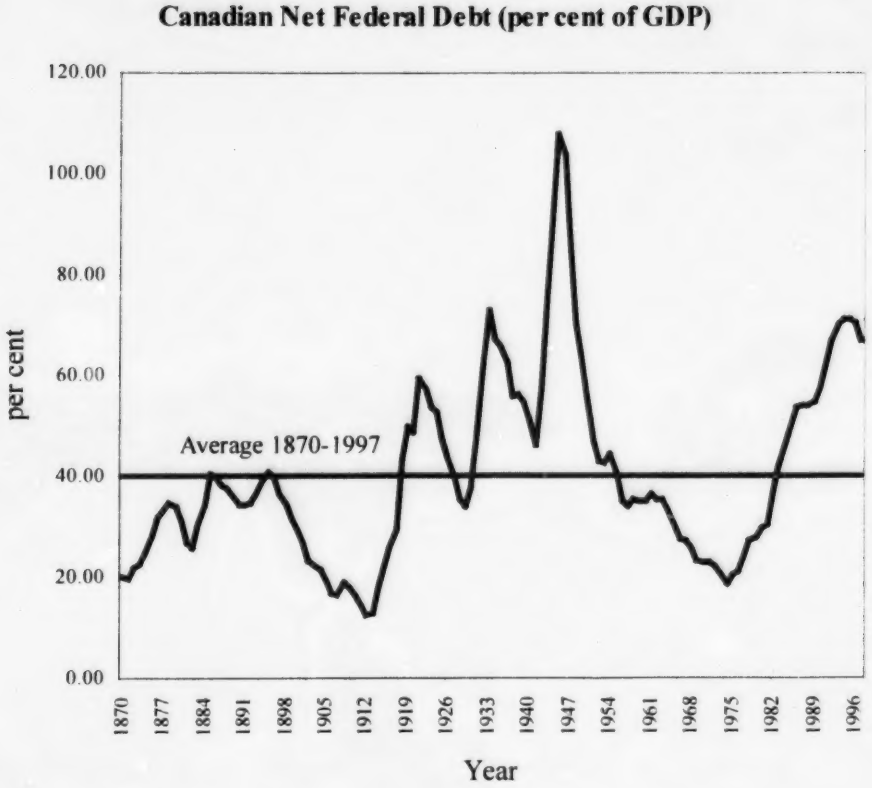
Our starting point is the seminal computable general equilibrium model developed by Aiyagari and McGrattan (1998) to estimate an optimal debt-to-GDP ratio for the United States. This model has the desirable feature of incorporating the effects of incomplete markets in a quite sophisticated way. This paper evaluates and extends the Aiyagari and McGrattan model and applies it to Canada.

2. The Standard Complete Markets Framework

Complete markets models have provided us with a firm understanding of the costs of government debt. Higher debt reduces national saving which leads to increased net foreign indebtedness and a lower capital stock. These effects in turn lead to lower output and income. Higher debt servicing costs force governments to raise distorting taxes and/or cut productive government spending. Higher debt reduces the government's capacity to stabilize the economy and engage in tax smoothing in the face of economic shocks. In this type of model, eliminating all government debt would be beneficial, and building up a large accumulated surplus would be better still.

Carrying this exercise to its logical conclusion would yield an 'optimal' policy, of building an accumulated surplus sufficiently large to allow all distorting taxes to be eliminated by financing expenditures entirely from interest income. This would imply a positive net financial asset position for the government of about 150 per cent of GDP. This would be optimal because it would eliminate all distortions and move the economy to its modified golden rule path.

The notion that a large accumulated surplus is optimal is unconvincing. It flies in the face of the fact that no government has ever pursued such a policy. Positive debt-to-GDP ratios have been the historical norm for all industrialized market economies. Canada has never had a negative net federal debt, and has averaged a net federal debt-to-GDP ratio of 40 per cent since confederation (see the Chart below).



If there really were large benefits from eliminating the debt and even accumulating surpluses, why has no government exploited these benefits? A possible answer is that while complete markets models are very good at capturing the costs of debt, they may be missing some important benefits of debt. The empirical shortcomings of complete markets models provide useful clues in this regard. It is well known that individual consumption, wealth and portfolio allocation data are inconsistent with the predictions of complete markets models. Incomplete markets models are better able to overcome these problems.

3. The Incomplete Markets Framework

In the incomplete markets literature, the economy is usually characterized by individuals who are subject to idiosyncratic and uninsurable shocks and by frictions in asset markets taking the form of borrowing constraints, costs of monitoring individual credit-worthiness or transactions costs. As a result, individuals typically face higher borrowing than lending rates and may be completely liquidity constrained when hit by negative

income shocks. Their inability to insure themselves against such will lead them to engage in precautionary saving.

Empirical Appeal

The incomplete markets approach generates the empirically plausible notion that individuals face substantially greater uncertainty than that represented by fluctuations in aggregate variables. Individual uncertainty is greater because people are unable to fully insure against idiosyncratic risks. Incomplete markets models are empirically appealing on a number of grounds:

- Many individuals face significant differentials in lending and borrowing rates that are consistent with imperfect monitoring of credit worthiness and the existence of transactions costs.

- Incomplete credit markets can address the equity premium and low risk free rate puzzles in the capital asset pricing literature.

- Consumption is more sensitive to disposable income than predicted by the permanent income hypothesis. Liquidity constraints would generate this result, although other explanations such as myopia would do the same.

Implications for Government Debt

Government debt can enhance the liquidity of private agents in a number of ways depending upon the underlying features of the particular incomplete markets model used. Debt may provide a relatively safe saving instrument to agents that could not otherwise be available. In this case gross debt enhances liquidity. On the other hand, debt may raise the overall private net asset positions of the agents even if government debt is indistinguishable in its risk characteristics from private assets. In this case net debt enhances liquidity.

The latter effect arises in the computable general equilibrium (CGE) model developed by Aiyagari and McGrattan (AM) to estimate the optimal debt-to-GDP ratio for the United States. This model incorporates the effects of incomplete markets in a sophisticated manner and is the only CGE model to do so.

In the AM model agents face borrowing constraints which prevent them from fully insuring themselves against idiosyncratic shocks. To self-insure, agents hold more physical capital than they would need under full insurance. An increase in net government debt enhances household liquidity by increasing the supply of financial assets that can be used for consumption smoothing. The consequent reduction in consumption volatility is welfare enhancing. Moreover, government debt allows agents to reduce their "inefficient" holdings of physical capital, which reduces the marginal product of capital. It thus appears as if government debt raises the interest rate and "crowds out" physical capital. However, in this model, the reduction in physical capital is desirable.

Assessment of the Aiyagari-McGrattan Model

The Aiyagari and McGrattan model is in steady-state form, and is calibrated to U.S. data. Aiyagari and McGrattan estimate that the optimal US net government debt-to-GDP ratio is 0.66 (very close to the U.S. post-war historical average) but find that the social welfare cost of deviating from this optimum is quite small. However, there are a number of reasons for viewing these results cautiously:

i) Government absorption is treated as waste. This creates an artificial benefit channel for debt, since the Aiyagari and McGrattan fiscal dividend rule implies that government absorption falls as the steady-state level of debt rises. Effectively, one reason that debt is good because it forces governments to waste less money.

ii) The model is closed. Open economy features would tend to weaken the liquidity benefits of government debt. This is because higher government debt will lead to higher net foreign indebtedness, reducing the size of the domestic agent's financial asset increase and the consequent liquidity benefit.

iii) The tax smoothing and automatic stabilization capacity of debt is not captured. This capacity is likely greatest when the government net asset position averages near zero, as this minimizes net interest payment variability.

iv) The model ignores the fact that some government social programmes such as Employment Insurance do provide at least partial insurance against idiosyncratic labour earnings shocks. Such programmes would tend to reduce the need for government debt to enhance private liquidity.

v) The model may not fully capture the persistence of the idiosyncratic shocks that people face. Accounting for this would strengthen the liquidity benefit of debt.

4. Extensions to the Aiyagari and McGrattan Model

The paper limits itself to improvements to the Aiyagari and McGrattan model. To this end, points (i) and (ii) are addressed by introducing alternative fiscal rules, a welfare role for government spending and open economy features. The paper compares the resulting welfare effects of government debt with those obtained by Aiyagari and McGrattan. Unlike Aiyagari and McGrattan, we do not interpret the resulting social welfare maxima as the true optima on the grounds that items (iii) to (v) must also be addressed which might require the use of other models.

The extensions are:

- To test the importance of (i) we shut off the absorption channel and force taxes to bear the full brunt of fiscal adjustment. This significantly raises the net cost of debt.

- The welfare role of government absorption is modelled and the Aiyagari and McGrattan fiscal dividend rule reintroduced. This moderately raises the net cost of debt.

- Open economy features are introduced. We assume imperfect substitutability between foreign and domestic assets, which implies home country asset preference. This may be regarded as intermediate between the polar extremes of a closed and small open economy. This also moderately raises the net cost of debt.

5. Conclusions

Aiyagari and McGrattan have identified an important channel for government debt that must be considered in determining an appropriate long-term benchmark. They are, however, too quick to claim to have found a precise estimate of the optimal debt-to-GDP ratio. The estimated optimum is quite sensitive to assumptions regarding the welfare effects of government absorption, the fiscal dividend rule and the openness of the economy.

While the specific optima are quite sensitive to these assumptions, the model does generate a qualitatively robust conclusion, namely, that the social welfare cost of government debt is less under incomplete markets than under complete markets. This is significant because it allows potentially greater scope for other factors such as the buffering role of debt in the determination of the overall optima.

Other implications are:

- Focussing fiscal dividends on the most welfare-enhancing government spending or the most distorting taxes implies a higher social welfare cost of government debt and a lower overall optimum debt-to-GDP ratio.

- A reduction in the ability of transfer programmes to insure individuals against negative idiosyncratic shocks implies a lower social welfare cost of government debt and a higher overall optimum debt-to-GDP ratio.

- Increased openness of the economy (defined as substitutability of domestic and foreign assets) implies a higher social welfare cost of government debt and a lower overall optimum debt-to-GDP ratio.

No single framework or model will suffice in determining an optimal debt policy. The extended AM model should be viewed as an important building block in broader work addressing the optimal debt-to-GDP ratio issue.

1. Introduction

Government debt generates well-understood costs. Higher debt reduces national saving which leads to increased net foreign indebtedness and a lower capital stock. These effects in turn lead to lower output and income. Higher debt servicing costs force governments to raise distorting taxes and/or cut productive government spending. Higher debt reduces the government's capacity to stabilize the economy and engage in tax smoothing in the face of economic shocks.

In this type of model, eliminating all government debt would be beneficial, and building up a large accumulated surplus would be better still. However, a growing literature argues that an important benefit of debt is its ability to overcome imperfections faced by individuals in asset markets. Borrowing constraints that prevent individuals from using their entire future labour income as collateral are an example of such an imperfection. Individuals with low levels of financial assets may be unable to use their human wealth (the present discounted value of their expected labour income) to buffer against labour income shocks.

In a world of incomplete financial markets, government debt can enhance the liquidity of private agents in a number of ways. First, the issuance of government debt directly raises the supply of financial assets relative to human wealth, and therefore improves private agent liquidity. This liquidity-enhancing effect occurs even if government debt is indistinguishable in its risk characteristics from claims on private firms; that is, it does not rely on any special characteristics of government bonds. All that is required is that the net government supply of financial assets increases. An increase in gross debt that was matched by higher gross asset holdings would not raise the net supply of financial assets available to private agents. This liquidity channel is of particular interest because net debt levels are generally the primary concern of policy-makers. The costs described earlier are also a function of net rather than gross debt thus there is a natural conceptual symmetry between those costs and this liquidity benefit. For these reasons, this paper focuses on this channel.

A second strand of the incomplete markets literature argues that government debt provides agents with a relatively safe saving instrument that would not otherwise be available. The liquidity service provided by treasury bills may be close to that of money, but with the advantage of a positive rate of return. In this case, gross debt enhances liquidity. While not the main concern of this paper, an "optimal" gross debt can ultimately have implications for net debt if the government has a desired gross asset position. For example, gross government asset holding may be limited by a desire to leave most of the economy in private hands for efficiency reasons.

This paper is organized as follows. Section 2 reviews effects of government debt in standard complete markets models. We then contrast this with the role of debt in incomplete markets models. We focus on a recent study by Aiyagari and McGrattan (1998), and provide an assessment of its strengths and weaknesses. Section 3 extends the

Aiyagari-McGrattan model by incorporating a variety of fiscal rules, a welfare role for government spending and open economy features. Section 4 presents conclusions, limitations and promising directions for further work.

2. Complete Versus Incomplete Markets

2.1 The Standard Complete Markets Framework

A number of computable general equilibrium studies have been undertaken using the complete markets framework to estimate the economic and social welfare effects of government debt. Examples include Auerbach and Kotlikoff (1987) for a closed U.S. economy and James (1994), James and Matier (1997) and Macklem, Rose and Tetlow (1994) for an open Canadian economy. In these models, debt generates costs via national saving and tax distortion channels.

Issuing debt, or running a deficit, represents a delaying of taxes until the future. For this to be costly, future taxes must be seen by agents as in some sense different from taxes today. If the future taxes associated with forgoing taxes today, or running a deficit, are seen as equal in present value to the current deficit, and if they are also non-distorting, or lump-sum, then no costs will follow. The notion that the debt-lump-sum tax mix might be irrelevant is called 'Ricardian equivalence' and has received renewed attention during the past fifteen years. Bernheim (1987) conducts an extensive literature review and argues that complete Ricardian equivalence is theoretically highly improbable and is not supported empirically.

For a given level of government expenditure, debt also imposes economic costs because of the use of distorting tax instruments to finance expenditures. This is conceptually separate from Ricardian equivalence per se, which asserts merely that the debt-lump-sum tax mix is irrelevant. As long as real capital returns exceed real growth rates, an increase in the debt-to-GDP ratio will imply that future distorting taxes will be higher, which will generate economic costs. Results in James (1994) and James and Matier (1997) suggest that the tax channel costs are greater than those generated by the dissaving channel.

These studies conclude that debt reduction leads to a higher capital stock, lower net foreign indebtedness (in the open economy cases) and lower distorting taxes. Debt reduction also generates transitional costs in the form of reduced consumption and output, however the studies find that these costs can be more than offset by the long run benefits of reduced debt.

In the standard framework, reducing the debt-to-GDP ratio from 100 per cent to 0 would be beneficial, but so would be a move from 0 to an accumulated surplus of 100 per cent. Carrying this exercise to its logical conclusion would yield an 'optimal' policy, namely building an accumulated surplus sufficiently large to allow all distorting taxes to be eliminated and expenditures to be financed entirely from interest income. This result is found in a number of studies in the public finance literature, for example, Barro (1979) and Chamley (1985) and (1986). At the federal level, it would imply an accumulated surplus of about 150 per cent of GDP. In the standard framework such a policy would be

optimal because it would eliminate all distortions and move the economy to its modified golden rule path --provided that the economy is 'dynamically efficient', that is, a higher capital stock would raise consumption per person. The fact that capital returns generally exceed economic growth rates suggests that the Canadian economy is dynamically efficient.

The notion that a large accumulated surplus is optimal flies in the face of the fact that no government has ever pursued such a policy. Indeed, it is extremely rare for any government to actually eliminate its debt, even during the 19th century when the size of government was small and fiscal profligacy was particularly frowned upon. The United States federal government eliminated its debt in only two years - 1835 and 1836, and Canada has never done so.

2.1.1 Empirical Shortcomings

The counterfactual features of the complete markets framework have been widely documented in the literature. The behaviour of individual consumption, wealth, and portfolios is strongly at variance with the complete markets models. These models fail to reconcile intertemporal asset pricing theory with historical observations on the joint behaviour of actual consumption and asset returns.

Real stock returns, real short government bond returns and the growth rate of per capita real consumption exhibit behaviour that give rise to the equity premium puzzle and the low risk free rate puzzle. Mehra and Prescott (1985) show that simple representative agent models (which assume complete and frictionless financial markets) imply excess returns on equity that are substantially smaller than those observed historically. The equity premium puzzle is that stocks are not sufficiently riskier than bonds to explain the spread in their returns[1]. The risk free rate puzzle is that individuals save more than a representative agent model would predict given low observed aggregate consumption variability and low risk free rates.

Underlying the Mehra and Prescott model is the presumption that the behaviour of per-capita consumption growth is a good guide to the behaviour of individual consumption growth. If asset markets are complete, individuals can write contracts against any contingency and idiosyncratic risks are easily diversified away. All consumption streams will look similar to each other and to per-capita consumption. In this environment, the equity premium puzzle arises from the fact that per-capita consumption is very smooth, and therefore its covariance with stock returns is small.

Realistically, trade cannot be made contingent upon the realization of all possible individual shocks and so most economists believe that insurance markets are incomplete in one way or another. The absence of complete insurance markets generates substantial risk at the individual level and in return, makes agents more hesitant to hold risky assets. This additional risk faced by the individual means that his consumption growth is more

---

[1] See Burnside and McCurdy (1992) for a literature review on the equity premium puzzle.

variable than per-capita consumption growth. Market incompleteness means that the additional variability in individual consumption is likely to generate a larger covariance of individual consumption growth with stock returns. Individuals must save more under incomplete markets in order to smooth random fluctuations in their consumption streams. This precautionary saving pushes the risk free rate below the complete markets interest rate.

2.2 The Incomplete Markets Framework

In incomplete markets models individuals face uninsurable idiosyncratic shocks. Asset markets feature frictions that can take the form of borrowing constraints, costs of monitoring individual credit-worthiness or transactions costs. Individuals may face higher borrowing than lending rates and may be completely liquidity constrained when hit by negative income shocks. Their consumption will consequently be more volatile than aggregate consumption.

2.2.1 The Role of Government Debt

The liquidity-enhancing role of debt can take different forms depending upon the underlying features of the particular incomplete markets model used. While models with market imperfections may yield results that are very different from their complete markets counterparts, not all market imperfections yield similar welfare results. As noted in the introduction, some models have implications for net debt while others have implications for gross debt. Debt may raise the overall private net asset positions of the agents even if government debt is indistinguishable in its risk characteristics from private assets. In this case, net debt enhances liquidity. On the other hand, government debt may provide a relatively safe saving instrument to agents that could not otherwise be available. In this case, gross debt enhances liquidity.

2.2.2 Models in which Gross Debt Enhances Liquidity

Bohn (1997) analyzes the effects of government debt and deficit financing strategies in a situation with low real interest rates, using an intergenerational equity criterion. He shows that the welfare assessment of persistent government deficits depends on what explains the low risk-free rate puzzle mentioned earlier. Bohn develops a model in which the government can issue debt at an interest rate below the interest rate on private loans and thus, increasing government debt reduces intermediation costs.

Hölmstrom and Tirole (1998) also examine the liquidity effects of government debt. They show how public debt can aid the corporate sector meet its liquidity needs. They address a basic, yet unresolved, question: do private assets provide sufficient liquidity for the efficient functioning of the productive sector? Liquidity here refers to the availability of instruments (market and nonmarket) that can be used to transfer wealth across periods. Typically, studies that have addressed the sufficiency of liquidity have focused on the consumer sector; however, Hölmstrom and Tirole build a model in which firms have liquidity needs. In order to protect against liquidity shocks a firm may want to hold liquid reserves in the form of, marketable assets that can be readily sold or by arranging in

advance for a line of credit (a long-term loan). The same agency problem that limits the amount of financing that firms can raise ex-post as a result of a negative liquidity shock also limits the amount of liquid assets that are available in the market to back up long-term financial commitments by outside investors. When firms can sell outsiders only a small fraction of their expected returns, the security market as a whole has a very limited capacity to transfer wealth from one period to the next.

Under aggregate uncertainty, the private sector cannot fully satisfy its own liquidity needs thus creating a potential demand for government-provided liquidity. Hölmstrom and Tirole show that the government can achieve a Pareto improvement by issuing Treasury bonds and that these bonds can be sold at a liquidity premium. The productive sector is willing to purchase low-yielding securities because they serve as an input to the production process.

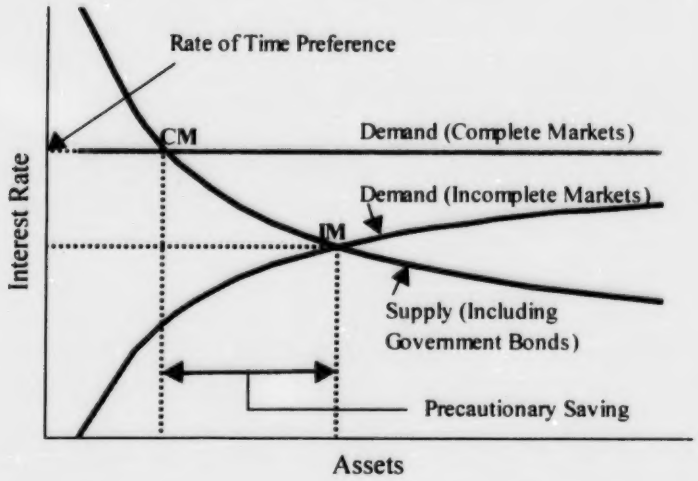
2.2.3 A Model in Which Net Debt Enhances Liquidity: The Aiyagari-McGrattan Model

The Aiyagari-McGrattan (1998) model features a continuum of infinitely lived agents who face uninsurable idiosyncratic labour earnings shocks and borrowing constraints, with no aggregate risk. The absence of complete markets combined with borrowing constraints leads agents to engage in precautionary saving in order to insure against these shocks.

A key feature of the Aiyagari and McGrattan model is that the equilibrium interest rate is lower, and the capital stock higher, than under complete markets. This is demonstrated graphically in Chart 1. This chart shows the aggregate supply and demand curves for financial assets (claims on private physical capital plus government bonds). The supply of physical assets is obtained by equating the marginal product of capital with the interest rate. In a world of complete markets, the demand curve is the horizontal line that intersects the interest rate axis at the rate of time preference. Equilibrium occurs at the point "CM".

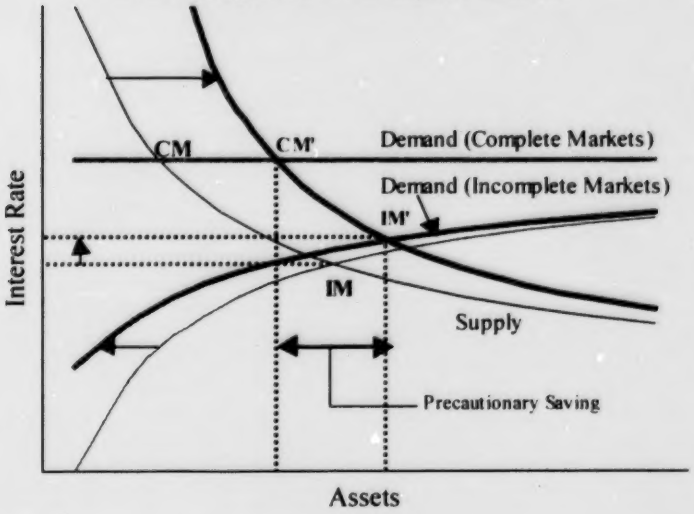
In contrast, under incomplete markets, asset demand would be infinite at this interest rate. If the interest rate equalled the rate of time preference, the agent would attempt to maintain a smooth marginal utility of consumption profile. At the margin, holding an additional unit of the asset is costless to the agent. However, given that there is a positive probability of receiving a long string of bad draws of labour income shocks, maintaining a smooth marginal utility of consumption would only be possible if the agent held an infinitely large amount of assets to buffer the shocks. Only if the interest rate is less than the rate of time preference is asset demand finite, because holding an additional unit of the asset is then costly. The asset demand curve is therefore upward sloping. The equilibrium is labelled "IM". Under incomplete markets the equilibrium interest rate is lower and the capital stock higher than under complete markets. The larger capital stock is the result of precautionary saving as indicated in the chart.

Chart 1: Equilibrium Under Complete and Incomplete Markets



Under incomplete markets, an increase in government debt enhances the liquidity of households by increasing the supply of financial assets that can be used for consumption smoothing. The consequent reduced need for precautionary saving means that the asset demand curve shifts to the left. Higher debt increases the interest rate and crowds out some physical capital, although total (including government) financial asset holdings increase. In contrast, under complete markets and infinite horizons, an increase in government debt has no effect on either the equilibrium interest rate or capital stock (see Chart 2). Aiyagari and McGrattan note that Ricardian equivalence breaks down under incomplete markets because an agent who becomes borrowing constrained is effectively bankrupt and subsequently reborn as a new economic actor. A positive probability of bankruptcy shortens effective planning horizons in a similar fashion to the positive household death and birth rates of the Buiter-Weil-Blanchard models.

Chart 2: Impact of a Government Debt Increase Under Complete and Incomplete Markets



2.2.4 Intuition Underlying the AM Result

To understand the intuition underlying the AM result, consider an isolated individual (e.g., Robinson Crusoe) subject to exogenous productivity shocks (e.g., due to the weather). To self-insure against the possibility of a string of bad productivity shocks, Crusoe's only option is to keep more extra physical capital (e.g., more seed corn) on hand. Relative to the case in which productivity insurance is available, Crusoe's capital stock is "too high" and the marginal product at capital (the interest rate) is "too low".

Ideally, Crusoe would like to buy productivity insurance, instead of keeping too much capital. But even if he could find other people in situations similar to his, the market for productivity insurance would probably fail because of moral hazard or adverse selection problems.

Now suppose that there is a government that issues net debt. Crusoe (and his counterparts) can buy this debt and use it as the store of their precautionary savings, selling their share of the debt when times are bad. The government debt serves as a (somewhat imperfect) substitute for productivity insurance. Individuals are able to reduce their physical capital holding, raising the marginal product of capital and therefore the interest rate.

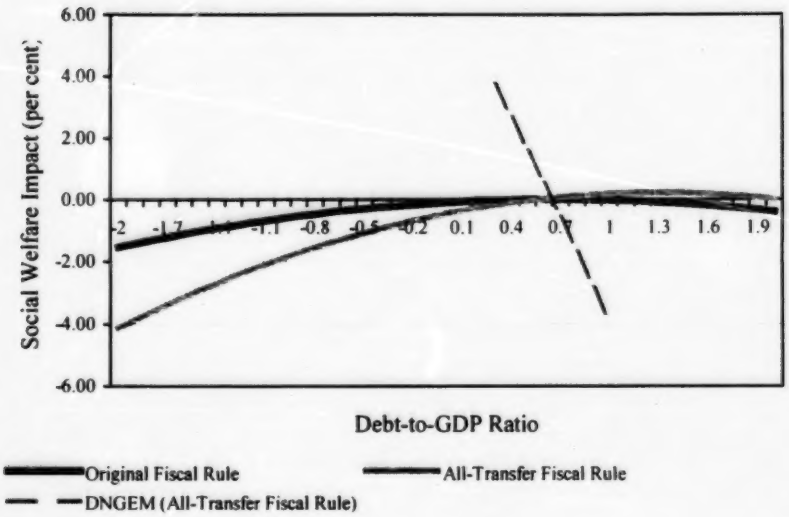
Note that private financial debt could not play the same role, because it must be secured by private physical capital, leading back to the problem of too much capital. The same problem would arise if the government issued gross debt backed by purchasing of shares in private capital. Government net debt is secured by the power of the state to enforce taxes. This allows the government to avoid the moral hazard and adverse selection problems that impede the provision of private productivity insurance.

2.2.5 Fiscal Policy in the AM model

The Aiyagari and McGrattan model is in steady-state form and is calibrated to U.S. data. The authors employ a fiscal rule that holds government spending and transfers constant as a proportion of GDP with income taxes adjusting residually as the steady-state debt-to-GDP ratio is adjusted. The authors estimate that the optimal debt-to-GDP ratio is .66 and that the curvature of the welfare profile over the entire region of positive debt-to-GDP ratios is quite flat (see Chart 3). Reducing the debt-to-GDP ratio from .66 to 0 implies a welfare loss of less than one-tenth of a percentage point in terms of consumption units.

Aiyagari and McGrattan simulate an alternative fiscal rule in which lump-sum transfers rather than distorting income taxes perform the residual adjustment. The resulting optimal debt-to-GDP ratio is 1.4, however the welfare profile remains quite flat (see Chart 3). The optimal debt-to-GDP ratio is higher in this case because the distorting tax cost channel has been removed, leaving the liquidity benefit channel to be balanced only by the savings cost channel. In contrast, simulations conducted using the complete markets Dynamic Neoclassical General Equilibrium Model (DNGEM) yield a high net social welfare cost of debt (see, e.g., James, 1994).

Chart 3: Original Aiyagari and McGrattan Model and DNGEM Results



2.2.6 Assessing the AM model

The Aiyagari and McGrattan model quantifies the liquidity benefit role of debt in an innovative and sophisticated manner. However, their claim to have precisely estimated the optimal debt-to-GDP ratio for the United States is premature for a number of reasons.

First, the model's treatment of government absorption combined with the particular fiscal dividend rule used tends to bias up the net benefit of debt. As the steady-state debt-to-GDP ratio increases, government absorption and transfers are allowed to move in proportion to changes in output, with taxes adjusting residually (implicitly reducing the tax cost channel) to maintain debt sustainability. Now, higher government debt typically implies lower long-run output which, given this rule, implies lower absorption and transfers. Absorption is treated as government waste; thus lower absorption is welfare enhancing. Increased debt can thus be welfare enhancing not only for liquidity reasons, but also because it forces governments to ultimately reduce wasteful expenditures. The welfare benefits of government spending would need to be modelled explicitly for the true welfare effects using this fiscal rule to be accurately judged.

Second, there is no aggregate uncertainty, hence there is no aggregate buffering role for debt. Optimally, the government net asset position acts as a buffer against economic shocks and lumpy government capital expenditures. This buffering role is desirable because of the superiority of smooth over fluctuating tax rates and because automatic stabilizers can help insulate the economy against demand shocks. If the government's objective is to bound the expected standard deviation of the debt-to-GDP ratio, then maximum buffering may be achieved by targeting an average net asset position of zero, as this will minimize the standard deviation of net interest payments.

Third, the ability of transfer programmes to provide partial insurance against idiosyncratic labour earnings shocks is ignored. Such programmes would decrease the liquidity-enhancing role of debt and the need for precautionary savings.

Fourth, the persistent nature of many idiosyncratic shocks may not be fully captured. Accounting for this would strengthen the liquidity benefit of debt. Transfer programmes may also lose their insurance capacity in the face of persistent negative shocks. For example, Employment Insurance benefits cannot be claimed for an indefinite period.

Fifth, Aiyagari and McGrattan assume a completely closed economy. This may not be a major problem for analysis of the United States, however, it is clearly not tenable for Canada. Open economy considerations are described in detail in Section 3.3.

A less serious problem is that transitional effects of changing the debt-to-GDP ratio are not captured. Typically, reducing the debt-to-GDP ratio leads to lower consumption initially, followed by higher consumption in the long run. However, the initial social welfare effect of a given debt reduction may simply be a scaled-down version of the steady-state social welfare effects, in which case transitional effects can be safely ignored in determining the optimal debt-to-GDP ratio.

3. Our Approach

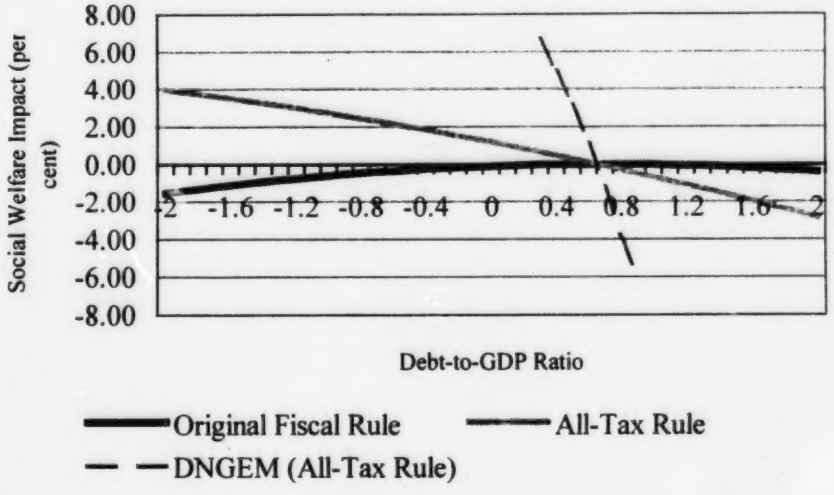
Although the Aiyagari and McGrattan results should be viewed cautiously, their model is nevertheless a very useful starting point. Our approach is to modify their model where possible in order to obtain more realistic results. To this end we consider an alternative fiscal rule, model the welfare benefits of government absorption, and introduce open economy features. The introduction of life cycle saving motives, transitional effects and aggregate uncertainty is left for future work. For tractability reasons, other models may be required to deal with these issues.

3.1 Alternative Fiscal Dividend Rule

We initially maintain the assumption that government absorption is waste but introduce an alternative fiscal dividend rule that shuts off the absorption channel described above. We exogenize the levels of absorption and transfers and allow their respective ratios to output to adjust freely rather than being held fixed as in Aiyagari and McGrattan . Distorting income taxes now bear the full brunt of adjustment as the debt-to-GDP ratio is varied.

Chart 4 shows that this rule dramatically alters the welfare profile. Improvements in the net asset position are now beneficial well into the accumulated surplus zone, although again less so than in the complete markets Dynamic Neoclassical General Equilibrium Model (DNGEM) simulations. The liquidity benefit channel is now overpowered by a strong tax cost channel and the perverse absorption benefit channel has been shut off.

Chart 4: Original Aiyagari and McGrattan Model; Original Versus All-Tax Fiscal Rule and DNGEM Results



The Aiyagari and McGrattan fiscal rule may actually be more realistic than a rule in which the entire fiscal dividend goes to income tax cuts. However, use of the Aiyagari and McGrattan fiscal rule clearly requires modelling the welfare benefits of government spending.

3.2 Introducing a Welfare Role for Government Spending

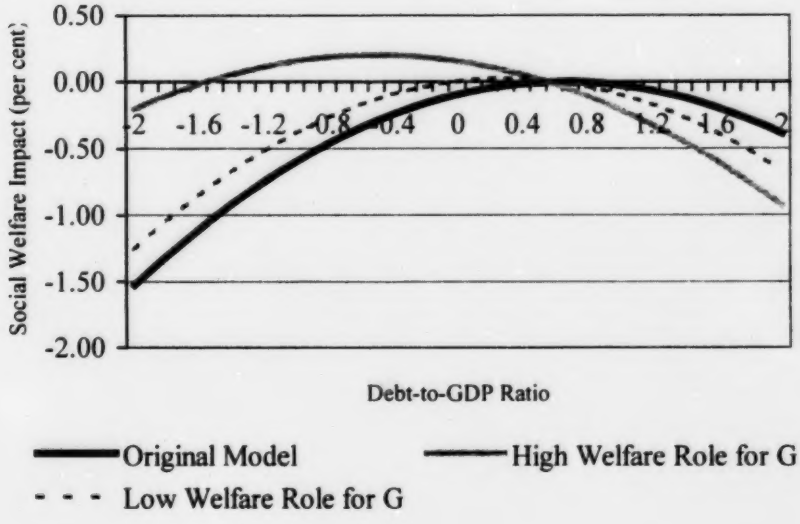
We assume that a portion ψ of government absorption is a perfect substitute for private consumption. Absorption thus enters private agents' utility functions as an exogenous add-on to private consumption. The momentary utility function thus takes the form:

$$u = \frac{\left[\left(c_t^{\eta} + \psi G_t\right)^{\eta} l^{(1-\eta)}\right]^{(1-\mu)}}{(1-\mu)}$$

where ψ is the marginal rate of substitution between private and public consumption goods, c is private consumption, G is government absorption per person, and l is leisure. We simulate our model for $\psi = 0.1$ (low welfare role) and 0.4 (high welfare role), which represents the range of admissible values calculated by Aschauer (1988).

Chart 5 shows that introducing a welfare role for government absorption raises the social welfare cost of debt, although by much less than the adoption of the all-tax fiscal rule. This follows by construction: the lower government spending associated with higher steady-state debt now directly reduces welfare.

Chart 5: Original Model Versus Welfare Role for Government Absorption: Original Fiscal Rule



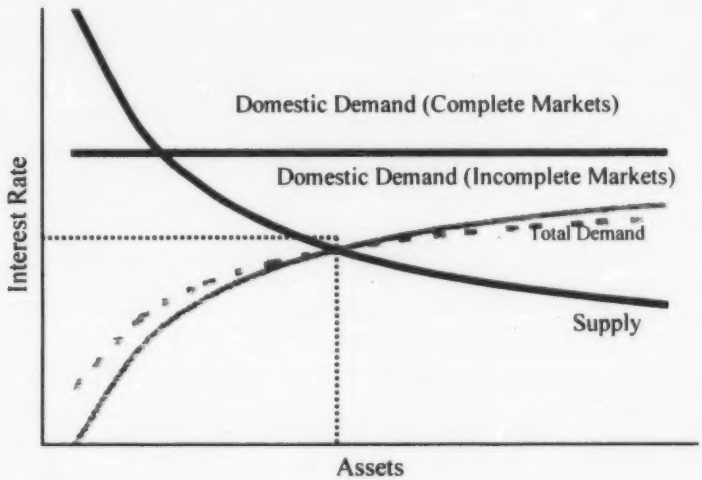
3.3 Introducing Open Economy Features

We modify the Aiyagari and McGrattan model to an open economy version featuring imperfect foreign-domestic asset substitutability resulting from home country asset preference[2]. Chart 6 demonstrates this case. The combined domestic and foreign demand for domestic assets is shown as the dotted curve (orange in the colour version). The combined demand curve is flatter than the domestic demand curve (shown as the solid curve alongside the combined demand curve, orange in the colour version) because an increase in the domestic-foreign interest rate differential raises the foreign demand for domestic assets. It would be completely flat if one assumed perfect substitutability of domestic and foreign assets.

Chart 7 shows that an increase in government debt raises the domestic interest rate, but by less than in the closed economy case. Net foreign indebtedness rises because the interest rate increase is not large enough to motivate domestic agents to hold all of the new supply of domestic assets. Total domestic asset holdings rise, as does domestic liquidity, but this effect is weaker than in the closed economy case owing to the net foreign indebtedness leakage.

The introduction of these open economy features thus weakens the liquidity benefit channel of government debt. The greater the openness of the economy, in the sense of a higher substitutability of foreign and domestic assets, the weaker this channel will be.

Chart 6: Open Economy Equilibrium Under Incomplete Markets



[2] See Goulder and Eichengreen (1988), (1992) and James (1994). Modelling details are provided in the Appendix.

Chart 7: Impact of Government Debt Increase in an Open Economy Under Incomplete Markets

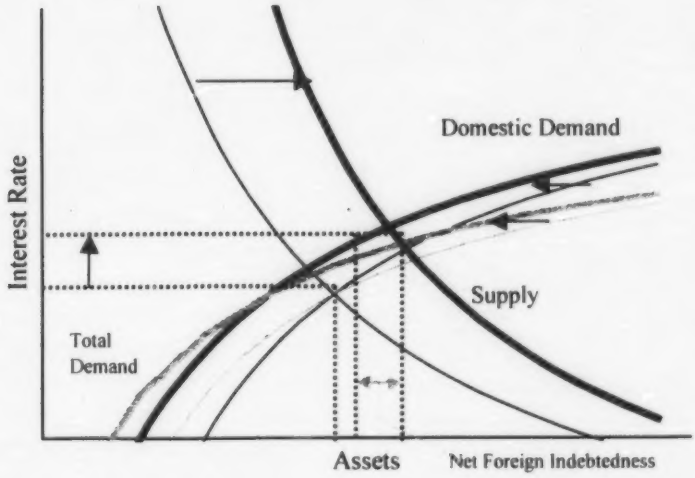


Chart 8 shows that, as expected, introducing open economy features raises the social welfare cost of debt. The cases in which a welfare role for government absorption are also included, may be regarded as the most complete, and therefore our most realistic cases.

Table 1 summarizes the simulation results by providing the steady-state social welfare impacts of reducing the net debt-to-GDP ratio from 65 to 15 per cent under various combinations of assumptions. Under incomplete markets, the net cost of debt is highest when the all-tax fiscal dividend rule is used, however even here the net cost is much less than that obtained from the complete markets DNGEM model.

Chart 8: Original Model Versus Open Economy Model (Canada); With and Without Welfare Role for Government Absorption; Original Fiscal Rule

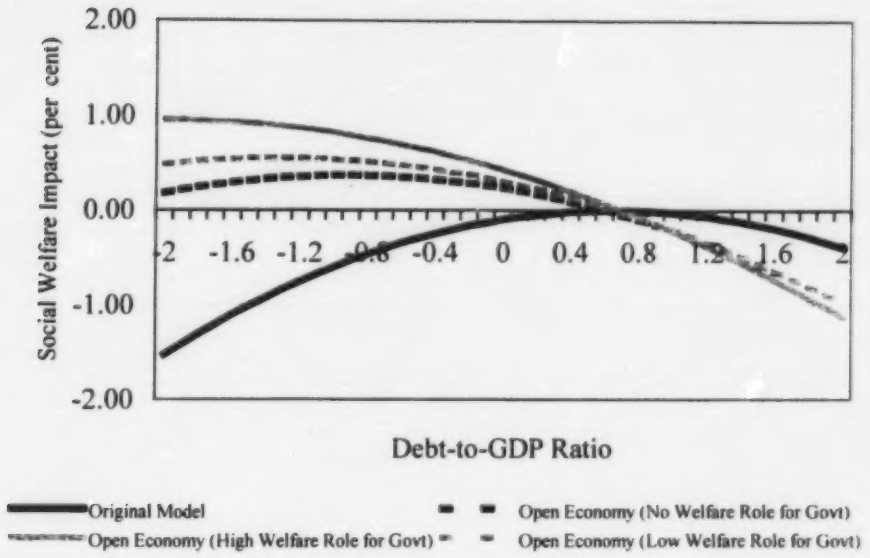


Table 1: Steady-State Social Welfare Impacts of Reducing the Net Debt-to-GDP Ratio From 65 to 15 Per Cent

Financial Markets	Fiscal Dividend Rule	Welfare Role for Government Spending	Open/Closed Economy	Steady-State Social Welfare Impact
Complete*	Proportionate Changes in All Taxes	None	Open (Imperfect Substitutability)	9.20
	Transfers			5.30
Incomplete**	Government Spending Proportionate to GDP; Taxes Adjust Residually		Closed	-0.06
	Proportionate Changes in All Taxes			0.88
	Transfers			-0.34
	Government Spending Proportionate to GDP; Taxes Adjust Residually	High		0.14
		Low		0.0005
		None	Open (Imperfect Substitutability)	0.21
		High		0.32
		Low		0.24

* DNGEM Simulation ** Aiyagari and McGrattan Original and Extended Model Simulations

4. Conclusions

Aiyagari and McGrattan have identified an important liquidity benefit channel for government debt that should be considered in determining an appropriate long-term benchmark. However, their estimate that the optimal U.S. debt-to-GDP ratio is .66 is premature. This estimate is quite sensitive to their assumption that government absorption is waste, the fiscal dividend rule they employ and the assumption of a closed economy.

The results of our extended model imply that focussing fiscal dividends on the most welfare-enhancing government spending or the most distorting taxes implies a higher social welfare cost of government debt and a lower overall optimum debt-to-GDP ratio. The results also show that increased openness of the economy (defined as substitutability of domestic and foreign assets) implies a higher social welfare cost of government debt and a lower overall optimum debt-to-GDP ratio. While we do not test this directly, it is apparent that a reduction in the ability of transfer programmes to insure individuals against negative idiosyncratic shocks implies a lower social welfare cost of government debt and a higher overall optimum debt-to-GDP ratio.

While the specific optima are quite sensitive to these assumptions, the model does generate a qualitatively robust conclusion, namely, that the social welfare cost of government debt is less under incomplete markets than under complete markets. This flatness of the social welfare profile is significant because it allows potentially greater scope for other factors, such as the buffering role of debt in the determination of the overall optima.

It would be useful to extend the Aiyagari and McGrattan model by adding a persistent component to the idiosyncratic shocks faced by agents. Panel data suggest that the variance of the persistent component of individual-specific income shocks is twice as large as that of the transitory component. Persistent shocks are less insurable and more likely to induce precautionary saving. The presence of persistent shocks would increase the liquidity-enhancing role of debt. Evidence from Storesletten, Telmer and Yaron (1998) suggests that adding a persistent component to idiosyncratic shocks can have important effects in incomplete markets models.

The nature of the relevant idiosyncratic shocks may also differ by income class. Richer people are more likely to be concerned with higher rates of return that high levels of assets can command and with random capital gains. Poorer people, in contrast, are more likely to be affected by government support policies that guarantee minimum consumption levels and by changes in health and marital status. The quantitative importance of any of these features on the precautionary motives to save of households requires further investigation. Preliminary findings by Quadrini and Rios-Rull (1997) indicate that including them allows for a better quantitative account of the savings motives and hence, of the wealth differences across households.

Greater realism would be achieved by incorporating overlapping generations with finite-lived agents. This would capture life-cycle saving motives and incomplete

intergenerational altruism and increase the importance of the saving cost channel of debt. It would also allow us to determine the implications for optimal debt of the demographic structure of the population and of generationally redistributive programmes like the CPP.

The Aiyagari and McGrattan and enhanced incomplete markets models calculate steady-state solutions only. Since debt reduction also imposes transitional costs not captured by the model, the steady-state and initial social welfare effects may differ.

No single framework or model will suffice in determining an optimal debt policy. The extended AM model should be viewed as an important building block in broader work addressing the optimal debt-to-GDP ratio issue.

Bibliography

Aiyagari, Rao and McGrattan, Ellen R. (1998), "The Optimum Quantity of Debt", *Journal of Monetary Economics* **42**, 447-469.

Aschauer, D.A., (1988), "The Equilibrium Approach to Fiscal Policy," *Journal of Money, Credit and Banking* **20(1)**, 41-62.

Barro, R.J. (1979), "On the Determination of Public Debt", *Journal of Political Economy* **87**, 940-971.

Bernheim, B.D. (1987), "Ricardian Equivalence: An Evaluation of Theory and Evidence", *NBER Macroeconomics Annual*.

Bohn, Henning (1997), "Fiscal Policy and the Mehra-Prescott Puzzle: On the Welfare Implications of Budget Deficits when Real Interest Rates are Low", Working paper, UCSB: Forthcoming *Journal of Money, Credit and Banking*.

Burnside, Craig and McCurdy, Thomas H. (1992), "Equity Premium Puzzle", The New Palgrave Dictionary of Money and Finance, 771-73.

Chamley, C. (1985), "Efficient Taxation in a Stylised Model of Intertemporal General Equilibrium", *International Economic Review* **26**, 451-468.

Chamley, C. (1985), " Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives", *Econometrica* **54**, 607-622.

Goulder, L.H., and Eichengreen, B. (1988), "Savings Promotion, Investment Promotion, and International Competitiveness", in *Trade Policies for International Competitiveness*, R. Feenstra (ed.), Chicago: University of Chicago Press.

Goulder, L.H., and Eichengreen, B. (1992), "Trade Liberalization in General Equilibrium: Intertemporal and Inter-industry Effects", *Canadian Journal of Economics*, **25(2)**, 253-80.

Hölmstrom and Tirole (1998), " Private and Public Supply of Liquidity", *Journal of Political Economy* **106(1)**, 1-40.

James S. (1994), "Debt Reduction With Distorting Taxes and Incomplete Ricardianism: A Computable Dynamic General Equilibrium Model", in *Deficit Reduction: What Pain, What Gain?* Robson, W.B.P., and W.M. Scarth (eds.), C.D. Howe Institute Policy Study No. 23.

James, Steven and Matier, Chris (1997), "Benefits of Government Debt Reduction: A Dynamic CGE Analysis of Alternative Transition Paths and Steady States", *Department*

of Finance, Economic Studies and Policy Analysis Division, Canada. Manuscript, revised December.

Macklem, R.T., D. Rose and R. Tetlow (1994), "Government Deficits and Debt in Canada: A Macro Simulation Analysis", in *Deficit Reduction: What Pain, What Gain?* Robson, W.B.P., and W.M. Scarth (eds.), C.D. Howe Institute Policy Study No. 23.

Mehra, R. and Prescott, E.C. (1985), "The Equity Premium: A Puzzle", *Journal of Monetary Economics* **15(2)**, 145-61.

Storesletten K., Telmer C. and Yaron, A. (1998), "Asset Pricing with Idiosyncratic Risk and Overlapping Generations", Manuscript: Graduate School of Industrial Administration, Carnegie Mellon University. Revised version July.

Quadrini, V and Rios-Rull, J.V. (1997), "Understanding the U.S. Distribution of Wealth", *Federal Reserve Bank of Minneapolis Quarterly Review*, **21(2)**, 22-36.

Technical Appendix

Technical Appendix: Mathematical Description, Decisions Functions, Joint Distribution and Welfare Measure

1. Mathematical Description

In each period, each agent is endowed with one unit of time that he devotes to leisure (denoted l) and to an aggregate production technology. The productivity, or efficiency, of an agent's endowment of time is denoted by e. The labour productivity shock e evolves in an idiosyncratic fashion according to a stochastic process modelled as a univariate stationary AR(1) in logs:

$$\log e_t = \rho_e \log e_{t-1} + \epsilon_{et} \tag{1}$$

The innovation ϵ_{et} is i.i.d. and normally distributed with conditional mean equal to zero and conditional variance given by $\sigma^2_{\epsilon_e}$.

Output in this world is produced by an aggregate technology to which individuals rent their labour services and capital. The aggregate production function is neoclassical and Cobb-Douglas, $Y_t = f(K_t, z_t N_t) = K_t^{\alpha}(z_t N_t)^{1-\alpha}$ where K, N, and Y represent per-capita labour, capital and output respectively and α denotes capital share . z_t is a measure of labour-augmenting, exogenous, technical progress. We assume that $z_t = z(1+g)^t$, where g is the rate of technical progress. The model has a proportional income tax τ_y (levied on the sum of labour and interest income) and elastic labour supply. Labour is supplied elastically to permit richer distortionary effects of the proportional tax system. When we assume competitive product and factor markets, the after-tax wage and the interest rate are denoted by $\bar{w}_t$ and $\bar{r}$, respectively, where

$$\bar{w}_t = (1-\tau_y) z_t F_2(K_t, z_t N) = (1-\tau_y) w_t \tag{2}$$

$$\bar{r} = (1-\tau_y)(F_1(K_t, z_t N) - \delta) = (1-\tau_y) r \tag{3}$$

with no time subscripts on r or N as these variables are constant in a balanced growth equilibrium.

The agent's behaviour is described as follows. Let c, l, a denote an agent's consumption (of a single non-storable commodity) in period t, leisure time and beginning-of-period asset holdings, respectively. Let the total time endowment for the household be normalized to unity. Let TR denote lump-sum (government) transfer payments. Consumers are charaterized by their endowments and their asset holdings. We assume that all agents have time separable preferences and they all discount the future by the same factor. The agent's period utility is specified as a standard isoelastic function $(c^{\eta} l^{(1-\eta)})^{(1-\mu)}/(1-\mu)$

that is twice continuously differentiable, increasing and concave in c. μ denotes the relative risk aversion parameter. Preferences for a typical agent are represented by the expected value of a discounted sum of per period utilities, $E[\sum_{t=0}^{\infty}\beta^t u(c_t,l_t) \mid a_0,e_0]$ where the expectation is assumed to be conditional on initial assets, a_0 and initial productivity shock, e_0.

1.1 Market Arrangements: Available Assets and Trading

The agent's budget and borrowing constraints take the form,

$$c_t + a_{t+1} \leq a_t(1+\bar{r}) + \bar{w}_t e_t(1-l_t) + TR_t, \qquad c_t \geq 0, 0 \leq l_t \leq 1, a_t \geq 0 \tag{4}$$

Let G_t denote per-capita government consumption and B_t denote per-capita government debt. With government consumption, capital evolves according to $K_{t+1} = Y_t - C_t - G_t + (1-\delta)K_t$. The government budget constraint takes the form,

$$G_t + TR_t + rB_t = B_{t+1} - B_t + \tau_y(w_t N + rA_t) \tag{5}$$

Agents can trade in risk-free assets, namely government bonds and capital as the two vehicles for savings. Let A_t denote per-capita assets held by consumers. In a balanced growth equilibrium, the asset market clears,

$$A_t = K_t + B_t \tag{6}$$

We also require that the labour market clears, where per-capita demand for labour by the firm equals the per-capita supply of labour by agents, $N = Ee_t(1-l_t)$. With average productivity normalized to unity ($Ee_t = 1$), this becomes,

$$N = 1 - E(e_t l_t) \tag{7}$$

This is a steady state model with growth. Along a balanced growth path, w_t, Y_t, K_t, B_t and A_t will grow at rate g. It is convenient to normalize variables relative to output as a stationarity-inducing transformation; besides the idiosyncratic shock, there are no other time-varying features which affect the problem the agent faces. The transformed stationary variables are now denoted as, $\tilde{c}_t = \frac{c_t}{Y_t}$, $\tilde{a}_t = \frac{c_t}{Y_t}$, $\tilde{w}_t = \frac{w_t}{Y_t}$, $k = \frac{K_t}{Y_t}$, $b = \frac{B_t}{Y_t}$, $\tilde{a} = \frac{A_t}{Y_t}$. In Aiyagari and McGrattan (1998), $\gamma = \frac{G_t}{Y_t}$ and $\chi = \frac{TR_t}{Y_t}$, are assumed to be fixed ratios as G_t and TR_t are assumed to be a fixed fraction of per-capita output.

Recasting the problem in its transformed form, the consumer problem becomes,

$$\underset{\tilde{c}_t,l_t,\tilde{a}_{t+1}}{\text{Max}}\ U(c,l) = E\big[Y_0^{\eta(1-\mu)}\sum_{t=0}^{\infty}[\beta(1+g)^{\eta(1-\mu)}]^t(\tilde{c}_t^{\eta}l_t^{1-\eta})^{1-\mu}/(1-\mu) \mid \tilde{a}_0,e_0\big],$$

subject to,

$$\tilde{c}_t + (1+g)\tilde{a}_{t+1} \leq \tilde{a}_t(1+\bar{r}) + \bar{w}_t e_t(1-l_t) + \chi$$

$$\tilde{c}_t \geq 0, 0 \leq l_t \leq 1, \tilde{a}_t \geq 0, t \geq 0,$$

We incorporate the inequality constraints in the maximization problem by attaching a penalty function (in equations 11 and 12 below) when asset holdings dip below zero or leisure exceeds the unit endowment of time.

The government budget constraint transformed, using the market clearing rate of return on capital and the wage rate, the asset market clearing $A_t = K_t + B_t$ and the homogeneous of degree one property of the production function, becomes:

$$\gamma + \chi + (\bar{r} - g)b = \tau_y(1 - \delta k) \tag{5'}$$

The asset market equilibrium condition transformed is $\bar{a} = k + b$. The capital-output ratio can be written as a function of r, that is $k = K/Y = \kappa(r)$. Using $\bar{w} = w_t/Y_t = (1-\alpha)/N_t$,

$$\bar{w} = (1-\tau_y)(w_t/Y_t) = (1-\tau_y)(\bar{w}) = (1-\tau_y)\omega(r) \tag{8}$$

The steady state of this economy is characterized by an interest rate r^* and per capita labour input N^*, which solve

$$\bar{a}(r, N; \gamma, b, g, \chi) = \kappa(r) + b \tag{9}$$

$$\bar{\varphi}(r, N; \gamma, b, g, \chi) = N \tag{10}$$

where $\bar{a}(.)$ is the per-capita demand for assets and $\bar{\varphi}(.)$ is the per-capita effective labour supplied by households.

Because the decision rules are stationary, we can compute the functions for consumption, asset holdings and leisure, $c(x,i)$, $\alpha(x,i)$ and $l(x,i)$ respectively that satisfy the first-order conditions of the problem. Primes, when used denote next period's variables. $x' = \alpha(x,i)$ is the decision rule and the computational task is to find an approximation to it by applying the finite element method [1] .

The state of the economy is represented by (ζ). ζ is a measure [2] that represents the cross-sectional distribution for this economy. It is defined over an appropriate family of

---

[1] See McGrattan (1996).

[2] To describe the economy at a point in time, we need a description of the different properties of the agents, that is, the size of groups of people in each wealth(asset)-earnings group. One way, mathematically, to describe the joint distribution of shocks and asset holdings is through a probability measure denoted by ζ.

subsets of $S = (\tilde{\mathcal{Z}} \times \mathcal{A})$, where $\tilde{\mathcal{Z}}$ is the set containing idiosyncratic shocks and $\mathcal{A}$ is the set of possible beginning-of-period wealth (the value of bonds plus capital) realizations. In words, ζ is simply a distribution of agents across idiosyncratic shocks and wealth. Because the economy does not feature aggregate shocks, a cross-sectional law of large numbers can be used to ensure that, in any stationary equilibrium, the function ζ is time-invariant.

1.2 Equilibrium; Characterization

We are interested in solving a stationary stochastic equilibrium. In such stochastic models with incomplete markets, steady states with a lower interest rate than the rate of time preference have been shown to exist. Steady states are found by iterative procedures once the model is set in its recursive form of the standard utility maximization problem. A recursive competitive equilibrium consists of:

- the individual's value function V and decision rules x',

- a pair of pricing functions (after-tax) $\bar{r}$ and $\bar{w}$ for labour and capital services,

- a set of government policies G, τ_y and TR

such that

- the $\{V, x'\}$ solves the agent's problem,

- competitive factor prices (i.e., given by marginal productivities as expressed in (2-3) above),

- markets clear (asset, labour and product) and aggregate quantities result from individual decisions: $K + B = \int_S a d\zeta$, $N = \int_S (1 - l) e d\zeta$ and $G + K' + \int_S c d\zeta = Y + (1 - \delta)K$,

- the government budget equation is satisfied and finally

- the decision rule and the process for the shock generate the next period distribution of agents ζ according to the mapping: $\zeta(S_0) = \int_{S_0} \{\int_S \chi_{a'=g(i,x)} \pi[i'/i] d\zeta\} di' dx'$ that is the steady state condition that the economy not change over time –given today's measure, the decisions of the agents replicate that measure tomorrow.

The steps involved in solving the equilibrium are as follows:

– Assume that the finite-element algorithm computes approximately the decision rules $a(x,i)$ and $H(x,i)$ where H is the joint distribution of assets and the productivity shock.

– Next, describe an algorithm for computing (1) the equilibrium interest rate and (2) aggregate hours.

• Start with an initial guess for N

• With N fixed, apply a *bisection method* to calculate the asset market clearing interest rate.

• In setting the bounds for the interest rate, we start with some initial value of r, say (r_1) and solve for average asset Ea at r_1, $Ea(r_1)$. The other bound r_2 is extracted from setting $Ea(r_1) = \frac{\alpha}{r_2+\delta} + b$. If r_2 exceeds the rate of time preference λ, it will assume the value of λ. Once the bounds are set, we can begin the bisection.

• An initial guess for the interest rate is $r = 0.5(r_1 + r_2)$

As inputs we have: α, γ, b, β, δ, μ, η, g, τ_1, τ_2, $e(i)$, π. π is the Markov transition matrix.

The tax rate is backed out from the government budget constraint:

$$\tau_y = \frac{\gamma + \chi + rb - gb}{1 + rb - \delta\frac{\alpha}{r+\delta}},$$

with τ_y and a guess for N, we can determine after-tax interest rate $\bar{r} = (1 - \tau_y)r$ and after-tax wage rate $\bar{w} = \frac{(1-\tau_y)(1-\alpha)}{N}$.

Next we use $\bar{r}$, $\bar{w}$ to compute the finite element approximation of a. Then, we combine the decision rule with the Markov process for e_t to calculate the stationary joint distribution H. The finite element method is also used to approximate H. Subsequently, H is used to calculate the mean asset holdings $E\tilde{a}_t$. With an estimate for $E\tilde{a}_t$, we iterate on the asset market clearing condition: $E\tilde{a}_t = \frac{\alpha}{r+\delta} + b$ until it is satisfied to within some tolerance level; this results in an equilibrium interest rate.

Once the above condition is satisfied, we update our guess for N. The updating scheme is simply Newton Raphson,

$$N^{k+1} = N^k - f(N^k)/J(N^k) \qquad \text{where,}$$

$$f(N^k) = \sum_i \int e(i)(1 - l^*(x,i,\alpha))dH(x,i) - N^k$$

Next compute,

- value function which satisfies Bellman's equation

- social welare

$$\sum_i \int V(x,i)dH(x,i) \qquad \text{where,}$$

$V(x,i)$ is the optimal value function and $H(x,i)$ is the steady state joint distribution function of assets and productivity. The conversion of this welfare criterion to a welfare measure in units of consumption is described below.

2. Decision Functions

In computing the decision functions, the consumer chooses sequences of consumption, asset holdings and leisure to maximize the expected utility subject to budget and borrowing constraints. Because the decision rules are stationary, we can compute the functions $c(x,i)$, $\alpha(x,i)$ and $l(x,i)$ that satisfy the following first-order conditions:

$$\eta(1+g)c(x,i)^{\eta(1-\mu)-1}l(x,i)^{(1-\eta)(1-\mu)} = \beta(1+g)^{\eta(1-\mu)}\Big[\sum_j \eta(1+\bar{r})c(x',j)^{\eta(1-\mu)-1}$$
$$l(x',j)^{(1-\eta)(1-\mu)} + \epsilon min(\alpha(x,i),0)^2\Big] \qquad (11)$$

$$(1-\eta)c(x,i)^{\eta(1-\mu)}l(x,i)^{(1-\eta)(1-\mu)-1} = \eta\bar{w}e(i)c(x,i)^{\eta(1-\mu)}l(x,i)^{(1-\eta)(1-\mu)}$$
$$+ \epsilon min(1-l(x,i),0)^2\Big] \qquad (12)$$

$$c(x,i) + (1+g)\alpha(x,i) = x(1+\bar{r}) + \bar{w}e(i)(1-l(x,i)) + \chi \qquad (13)$$

$$x' = \alpha(x,i) \qquad (14)$$

for $i = 1,\ldots,m$ and $x \in [0, x_{max}]$. $\epsilon min(.)$ is associated with the penalty functions for the two sets of inequality constraints.

We apply a Newton method to find a numerical solution for $l(x,i)$, denoted by $l^*(x,i;\alpha)$. With some manipulation, we rewrite the first order conditions in terms of

one residual, $R(x,i;\alpha)$. The computational task is to find an approximation for $\alpha(x,i)$ -say $\alpha^h(x,i)$ that implies $R(x,i;\alpha^h) \approx 0, \forall x,i$.

3. Unique Stationary Joint Distribution over Assets and Productivities

Next, to compute the equilibirum prices in the economy, we must first construct the joint distribution over assets and productivity. In defining the stationary steady state equilibrium, we made use of the measure concept. A measure acts as a counting mechanism and this is needed to describe an economy, where at any point in time households differ in their current luck, i and in their wealth x. In their technical appendix, Aiyagari and McGrattan discuss the existence of an invariant distribution, $H^* = H_n = H_{n+1}$ which is the fixed point of the dynamic equation of the model that gives tomorrow's distribution of households as a function of today's distribution and the Markov chain [3] .

4. Calculating the Welfare Costs in Terms of Consumption

The welfare measure is the percentage increase in benchmark consumption at every date and state (with leisure at every date and state held at benchmark values) that leads to the same value of the welfare standard as its value at any alternative debt-to-GDP ratio. v denotes the welfare cost/benefit in terms of consumption.

$$V = E_0\Big[\sum_{t=0}^{\infty}\beta^t\left(c_t^{\eta}l_t^{1-\eta}\right)^{1-\mu}/(1-\mu)\Big] \tag{15}$$

$$= E_0\Big[\sum_{t=0}^{\infty}\beta^t[((1+v)c_t^{\circ})^{\eta}l_t^{\circ 1-\eta}]^{1-\mu}/(1-\mu)\Big]$$

where the superscript 'o' refers to the baseline relative to which the welfare cost or benefit is calculated.

$$V^{\circ} = E_0\Big[\sum_{t=0}^{\infty}\beta^t\left((c_t^{\circ})^{\eta}(l_t^{\circ})^{1-\eta}\right)^{1-\mu}/(1-\mu)\Big] \tag{16}$$

dividing (15) by (16),

$$\frac{V}{V^{\circ}} = (1+v)^{\eta(1-\mu)} \tag{17}$$

After we transform the variables by dividing by output, (15) and (16) are rewritten as,

$$V = Y_0^{\eta(1-\mu)}E_0\Big[\sum[\beta(1+g)^{\eta(1-\mu)}]^t(\tilde{c}_t^{\eta}l_t^{1-\eta})^{1-\mu}/(1-\mu)\Big] \tag{15'}$$

[3] See also Ríos-Rull (1997) for a proof of the existence of a unique fixed point to the dynamic equation (eq-5- in his paper). As long as a certain *monotone mixing condition* is satisfied, it can be proved that: H^* exists.

$$= Y_0^{\eta(1-\mu)}\tilde{V}$$

$$V^\circ = (Y_0^\circ)^{\eta(1-\mu)}E_0\left[\sum[\beta(1+g)^{\eta(1-\mu)}]^t((\tilde{c}_t^\circ)^\eta(l_t^\circ)^{1-\eta})^{1-\mu}/(1-\mu)\right] \qquad (16')$$
$$= (Y_0^\circ)^{\eta(1-\mu)}\tilde{V}^\circ$$

dividing (15') by (16'),

$$\frac{V}{V^\circ} = \frac{Y_0^{\eta(1-\mu)}}{(Y_0^\circ)^{\eta(1-\mu)}}\frac{\tilde{V}}{\tilde{V}^\circ} \qquad (18)$$

Substituting,

$$\frac{Y_0}{Y_0^\circ} = \left(\frac{r+\delta}{r_0+\delta}\right)^{\frac{\alpha}{\alpha-1}}\frac{L_0}{L_0^\circ}$$

into (18) and equating (18) with (17) yields the welfare cost/benefit in terms of consumption; $(v \times 100)$ is what we depict in the welfare graphs,

$$v = \left(\frac{r+\delta}{r_0+\delta}\right)^{\frac{\alpha}{\alpha-1}}\frac{L_0}{L_0^\circ}\left(\frac{\tilde{V}}{\tilde{V}^\circ}\right)^{\frac{1}{\eta(1-\mu)}} - 1 \qquad (19)$$

5. Open Economy Features

5.1 Small Open Economy Case

To extend the analysis to the case of a small open economy, we let FD be the economy net foreign debt position. In a small open economy, perfect capital mobility pegs the interest rate; the domestic interest rate is set by the world rate, denoted r^w [4] . The economy-wide resource constraint becomes:

$$Y_t = C_t + G_t + K_{t+1} - (1-\delta)K_t + (1+r^w)FD_t - F_{t+1} \qquad (20)$$

The government budget constraint takes the form,

$$G_t + TR_t + r^w B_t = B_{t+1} - B_t + \tau_y(w_t N + r^w A_t) + \tau_y r^w B_t \qquad (21)$$

where $r^w = r^{w'}/(1-\tau_y)$ [5] . Transformed, (21) is written as,

$$\gamma + \chi + r^{w'}b - gb = \tau_y\left(1 - \delta\frac{\alpha}{\frac{r^{w'}}{1-\tau_y}+\delta}\right) \qquad (21')$$

---

[4] In our analysis, the foreign sector behaviour is not explicitly shown. Its behaviour is implicitly captured by r^w. An extension to this can take foreign sector behaviour to be derived from an explicit optimizing setting rather than being imposed.

[5] $r^{w'}$ is calibrated to be 0.0267 based on a (U.S.) world rate of interest r^w of 0.04275 and a tax rate of 0.375.

where τ_y is backed out from this equation as the solution to a quadratic function.

The agent's budget constraint (transformed) takes the form,

$$\tilde{c}_t + (1+g)\tilde{a}_{t+1} \leq \tilde{a}_t(1+\tau^{w'}) + \tilde{w}_t e_t(1-l_t) + \chi$$

The per-capita asset market clearing condition (transformed), to be used in characterizing the steady state of this economy is now,

$$E\tilde{a}_t = \frac{\alpha}{\frac{r^{w'}}{1-\tau_y} + \delta} + b - fd \tag{22}$$

(k - fd) is domestically owned capital. The bisection method is now applied on the foreign debt component which yields satisfaction of the asset market clearing to within some tolerance parameter.

5.2 Imperfect Asset Substitutability

In this open economy case, households show a home-country preference in holding assets. The rate of interest is endogenized while fd is exogenous. In the asset market clearing condition $E\tilde{a}_t = \frac{\alpha}{r+\delta} + b - fd$, the foreign debt variable is set according to the following equation:

$$fd = \phi(b - b_0) \tag{23}$$

where ϕ denotes a plausible net-foreign-liabilities-to-GDP ratio in Canada and $(b - b_0)$ is the deviation of debt-to-GDP ratio from benchmark.

Choosing ϕ along with measured shares, q, q^F and q_0 is equivalent to calibrating ξ_1, ξ_2, ξ_1^F, ξ_2^F in:

$$V = E_0\Big[\sum_{t=0}^{\infty}\beta^t\{(c_t^{\eta}l_t^{1-\eta})^{\xi_1}\Omega^{1-\xi_1}\}^{1-\mu}/(1-\mu)\Big] \tag{24}$$

$$\Omega = \xi_3\{(1-q_0)^{1-\xi_2}(1-q)^{\xi_2} + q_0^{1-\xi_2}q^{\xi_2}\}^{\frac{1}{\xi_2}}$$

$$V = E_0\Big[\sum_{t=0}^{\infty}\beta^t\{((c_t^F)^{\eta}(l_t^F)^{1-\eta})^{\xi_1^F}(\Omega^F)^{1-\xi_1^F}\}^{1-\mu}/(1-\mu)\Big] \tag{25}$$

$$\Omega^F = \xi_3^F\{(1-q_0^F)^{1-\xi_2^F}(1-q^F)^{\xi_2^F} + (q_0^F)^{1-\xi_2^F}(q^F)^{\xi_2^F}\}^{\frac{1}{\xi_2^F}}$$

where q is the share of domestic assets in the foreign agent's portfolio, q^F is the share of foreign assets in the foreign agent's portfolio and q_0 is the share under zero asset substitutability. We consider the case of $\phi = 0.3$.

6. Calibration

The model is calibrated to a period of a year. For the parameters of the labour earnings process, we use a Markov chain specification with seven states to match the first-order autoregressive representation of the log of labour earnings as in (1) above [6] . For moderate sized state spaces, Aiyagari (1994) showed that the approximation error from this method of approximation with seven states is quite small –the specified (estimated) serial coefficient, ρ_e, and the standard deviation of the innovation, σ_{ϵ_e} closely approximate the ρ and σ values implied by the Markov chain.

The aggregate production function is Cobb-Douglas, with the capital share set equal to 0.3. We allow for a 7.5% annual depreciation rate on the aggregate capital stock. The secular growth rate in per-capita GDP (we normalize the individual quantities in the model by dividing them by GDP, Y), is taken to be 1.85%. We parameterize the period utility function with a standard isoelastic specification and set μ to 1.5. The technology and preference specifications and the parameter values are chosen to be consistent with aggregate features of the postwar U.S. economy. The utility discount factor is set to 0.991. We set $\gamma = 0.217$ and $\chi = 0.082$. We set the debt-to-GDP ratio to 0.67 for the benchmark economy to be used as a yardstick to compute deviations from it in assessing welfare costs. ρ_e is set to 0.6 and σ_{ϵ_e} is set to 0.3; they are in the range of estimates found in the literature (See Heaton and Lucas (1996)). η is set to 0.328 to yield a labour elasticity of 2.

[6] Simply see Tauchen's (1986) method for choosing values for the state variables and the transition probabilities so that the resulting finite-state Markov chain closely mimics the underlying continuous-valued autoregression.

2001
2001-20 Monetary Policy and Medium-Term Fiscal Planning - Douglas Hostland
2001-19 NAOMI A New Quarterly Forecasting Model, Part 1: Proposed Model Selection Strategy - Stephen Murchison
2001-18 The Contribuion of Labour Markets, Demogaphics and Public Redistribution to Inequality: A Decomposition Analysis for Canada, 1986-1996 - Marcelin Joanis and Edgard Rodriguez
2001-17 Oils Sans Tax Expenditures - Ken Ketchum, Robert Lavigne, and Reg Plummer
2001-16 The Role of Fiscal Rules in Determining Fiscal Performance - Suzanne Kennedy and Janine Robbins
2001-15 An Empirical Search for a Canadian Credit Channel - Joe Italiano
2001-14 Specification of a Stochastic Simulation Model for the Analysis of Monetary and Fiscal Policy - Doug Hostland
2001-13 An Opportunistic Approach to Debt Reduction - Patrick Georges
2001-12 An examination of alternative Strategies for Reducing Public Debt in the Presence of Uncertainty - Doug Hosltand and Chris Matier
2001-11 Does Under-Investment Contribute to the Canada-U.S. Productivity Gap? - Edgard Rodriguez and Timothy Sargent
2001-10 A Multi-Sector Multi-Country Dynamic General Equilibrium Model With Imperfect Competition - Claude Lavoie, Marcel Mérette, and Mokhtar Souissi
2001-09 The fair value of the U.S. stock market: A structural VECM approach - Céline Gauthier

2001-08 A Medium-Term Forecasting Equation for the Canada-U.S. Real Exchange Rate - Martin Charron

2001-07 Constructing NEO: A Near-term Employment Outlook - Edda Claus
NOT YET PUBLISHED

2001-06 Capacity Utilization and Inflation: Is Statistics Canada's Measure an Appropriate Indicator of Inflationary Pressures - Max Baylor

2001-05 Monitoring Regional Economies in Canada With New High-Frequency Coincident Indexes - Robert Lamy and Patrick Sabourin

2001-04 Labour or Total Factor Productivity: Do We Need to Choose? - Timothy C. Sargent and Edgard R. Rodriguez

2001-03 Structural Unemployment and Technical Change in Canada, 1990-1999 - Timothy C. Sargent

2001-02 Determinants of Retirement: Does Money Really matter? - Janice Compton

2001-01 The Role of Government Debt in a World on Incomplete Financial Markets - Steven James
and Philippe Karam (*Appendix available in paper copy only)*

2000

2000-08 Public Finance Implications of Population Ageing - Phillip King and Harriet Jackson

2000-07 Networks for Macroeconomic Forecasting: A Complementary Approach to Linear Regression Models - Steven Gonzalez

2000-06 Forecasting employment Rates: A Cohort Approach - Marie-France Paquet and Timothy C. Sargent

2000-05 Measuring Human Capital in Canada - Mireille Laroche and Marcel Mérette of the University of Ottawa

2000-04 L'écart à terme comme indicateur de l'activité économique : L'expérience des années 90 Paul Delâge et Jean-François Fillion

2000-03 Croissance endogène et vieillissement démographique : le cas d'une petite économie ouverte* Maxime Fougère

2000-02 "Modeling Federal Finances under Uncertainty" Derek Hermanutz and Chris Matier

2000-01 "Liability Management Using Dynamic Portfolio Strategies" Richard Black, Colin R. Telmer

1999

99-03 "An Econometric Examination of the Impact of Population Ageing on Personal Savings in Canada" , Maxime Fougère - Finance, Marcel Mérette - University of Ottawa

99-02 "Analyzing and Forecasting Credit Ratings: Some Canadian Evidence" Patrick Sabourin

99-01 "Taxation and Economic Performance: A Cross-Country Comparison and Model Sensitivity Analysis" Jing Xu

1998

98-09 "The BU Ratio: Prospect and Retrospect" Timothy C. Sargent

98-08 "Youth and the 1990s Labour Market" Carolyn Wilkins

98-07 "The Canadian Economic and Fiscal Model - 1996 Version: Part 3 - Empirical Specification and Statistical Assessment" Jian-Guo Cao and Benoît Robidoux

98-06 "The Canadian Economic and Fiscal Model - 1996 Version: Part 2 - Dynamic Forecasting and Simulation Properties" Alain DeSerres, Benoît Robidoux and Bing-Sun Wong

98-05 "The Canadian Economic and Fiscal Model - 1996 Version: Part 1 - Model Structure" Benoît Robidoux and Bing-Sun Wong

98-04 "Population Ageing and the Current Account in Selected OECD Countries" Maxime Fougère and Marcel Mérette

98-03 "Population Ageing and Economic Growth in Seven OECD Countries" Maxime Fougère and Marcel Mérette

98-02 "The Persistence of Low Income Spells in Canada, 1982-1993" Mireille Laroche

98-01 "On the Concept and Dimensions of Human Capital in a Knowledge-Based Economy Context" Mireille Laroche, Marcel Mérette and G.C. Ruggeri

1997

97-08 "Estimating the Economic Efficiency Consequences of Alternative Tax Reforms Using a CGE Model: Homogeneous Vs Heterogeneous Labour Specifications" Louis Beauséjour, Mokhtar Souissi, Carole Vincent and G.C. Ruggeri

97-07 "Tax Expenditures and Tax Preferences in the Personal Income Tax System of Selected OECD Countries: A Suggested Classification, G.C. Ruggeri and Carole Vincent

97-06 "Income Taxes, Life-Cycles and Growth", Marcel Mérette

97-05 "Effective Income Tax Rates in Macroeconomics: An Update for Canada", G.C. Ruggeri, Mireille Laroche and Carole Vincent

97-04 "A Public versus a Private Canada Pension Plan: A Survey of the Economics" Steven James

97-03 "Forecasting Canadian Recessions with Macroeconomic Indicators" Robert Lamy

97-02 "The Dynamic Effects of Taxes and Government Spending in a Calibrated Canadian Endogenous Growth Model" Jing Xu

97-01 "The Impact of Fiscal Policy on the Risk Premium of Government Long-Term Debt: Some Canadian Evidence" Todd Mattina and François Delorme

1996

96-06 "Does the Length of the Current Expansion Increase the Probability of Recession?" Lisa Backa

96-02 "Do Mechanical Filters Provide a Good Approximation of Business Cycles?", Alain Guay and Pierre St-Amant

96-01 "An Indicator Model of Core Inflation in Canada" Robert Lamy and Paul Rochon

1995

95-12 "Potential Economic Effects of Experience-Rating the Unemployment Insurance System Using a Multi-Sector General Equilibrium Model of Canada" Louis Beauséjour, Munir A. Sheikh and Baxter Williams

95-11 "Active Labour Market Policies: An Evaluation" Lori Marchildon
95-10 "An Index of Unemployment Insurance Disincentives" Timothy C. Sargent
95-09 "The Economics of Canada Pension Plan Reforms" Steven James, Chris Matier, Humam Sakhnini and Munir Sheikh
95-08 "The Long-Run Economic Impacts of Government Debt Reduction" Steven James and Chris Matier
95-07 "Time Series Properties of Canadian Real Interest Rates", Chris Matier and Werner Schüle
95-06 "Interpreting Sacrifice Ratios Across Countries and Over Time" Brenda Lipsett and Steven James
95-05 "Effective Rates of Assistance: An Old Idea Whose Time Has Come?" Louis Beauséjour and Munir A. Sheikh
95-04 "Aggregate Fluctuations, Monetary Policy and Nominal Wage Rigidities in a New-Classical Setting: Results for Canada" Humam Sakhnini
95-03 "Ricardian Equivalence and the Permanent Income Hypothesis: An Empirical Investigation" Benoit Perron
95-02 "Are New-Classical Models Becoming Relevant Policy Tools: A Survey of Recent New-Classical Models" Humam Sakhnini
95-01 "Fluctuations économiques dans un environnement de stabilité des prix" Claude Lavoie Please note that this working paper is available in French only.
1994
94-06 "Conditional Productivity Convergence in Canada" Frank C. Lee
94-05 "A Guide to Applied Modern Macroeconometrics" Alain Paquet
94-04 "Interindustry Wage Differentials: Evidence, Explanations and Potential Distortions" Mario Iacobacci
94-03 "Regional Productivity Convergence in Canada" Frank C. Lee and Serge Coulombe
94-02 "Regional Economic Disparities in Canada" Serge Coulombe and Frank C. Lee
94-01 "The Interaction of Inflation with a Canadian-Type Capital Tax System: A Dynamic General

	Equilibrium Analysis Incorporating Endogenous Leverage and Heterogeneous Households" Steven James
1992	
92-04	"An Environmental CGE Model of Canada and the United States" Louis Beauséjour, Gordon Lenjosek and Michael Smart
92-03	"Estimating Trend Total Factor Productivity Growth in Canada" Carolyn Wilkins, Frank Lee and Steven James
92-02	"Un indice de prix régional de biens et services comparables au Canada et son application aux disparités régionales de revenu" Raynald Létourneau. Available in French only.
92-01	"A New Composite Leading Indicator of the Canadian Economy" Robert Lamy
1990	
90-07	"Une analyse empirique des déterminants du taux d'intérêt réel mondial" Benoît Robidoux
	Please note that this working paper is available in French only.
90-06	"Real Interest Rates in Historical Perspective" Paul Jenkins
90-05	"The Contribution of Fiscal Policy to Real Interest Rate Movements in the 1980s" Paul Jenkins
90-04	"The Constant Price Hypothesis in Multi-Industry Cross-Section Studies: The Impact on Cost Function Estimates" Benoît Robidoux and John Lester
90-03	"The Cyclically-Adjusted Output Methodology" Mario Albert, Nathalie Boisvert and Robert Fonberg
90-02	"The Determination of Real Interest Rates" Joel Fried and David Burgess
90-01	"La voie des dépenses fiscales : le point sur les questions conceptuelles et les controverses"
	Neil Bruce. Available in French only.
1989	
89-07	"INTERMOD 2.0: Model Specification and Simulation Properties" Guy Meredith
89-06	"The Calculation of Marginal Effective Tax Rates in the 1987 White Paper on Tax Reform"
	Jack Jung
89-05	"Effects of Changing Age Structure on Consumption and Saving" Paul Jenkins

89-04 "Expectations, Policy Shocks, and Output Behaviour in a Multi-Country Macroeconomic Model"
Guy Meredith

89-03 "The Goods and Services Tax: A General Equilibrium Analysis" Bob Hamilton and Chun-Yan Kuo

89-02 "A Survey of Some Recent Developments in Macroeconomics" Michael Devereux

89-01 "Closed Economy Analysis of the Dynamics of Deficits and Debt in a Unitary and Federal State" - Carolyn Wilkins and Mario Albert

1988

88-10 "A Guide to the GET Model" Richard G. Harris

88-09 "INTERMOD Model User's and Developer's Guide for PC Systems" Philip Bagnoli and Anders Kruus

88-08 "The Trade Sector in INTERMOD" Guy Meredith

88-07 "INTERMOD 1.1: A G-7 Version of the IMF's Multimod" John F. Helliwell, Guy Meredith, Yves Durand and Philip Bagnoli

88-06 "Resolutions of the U.S. Trade Imbalances: How Painful Will the Adjust Be?" Janice Haritos

88-05 "Econometric Estimates of Import Price Elasticities for Canada" Raynald Létourneau and John Lester

88-04 "Econometric Estimates of Scale Economies in Canadian Manufacturing" Benoît Robidoux and John Lester

88-03 "Trade Barriers Between Canada and the United States" John Lester and Tony Morehen

88-02 "New Estimates of Canadian Tariff Rates by Industry and Commodity" John Lester and Tony Morehen

88-01 "Geographic Labour Mobility in Canada" Louis Grignon and Kei Moray

1987	
87-01	"Économies d'échelle : Un survol des techniques d'estimation et des problèmes méthodologiques"
	Benoît Robidoux. Available in French only.
1986	
86-01	"The Structure of Production in Ten Canadian Industries" François Delorme and John Lester